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06CS42

Fourth Semester B.E. Degree Examination, December 2012
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Seven towns a, b, c, d, e, f and g are connected by a system highways as follows:
   i) NH – 22 goes from a to c, passing through b.
   ii) NH – 33 goes from c to d and then passes through b as it continues to f.
   iii) NH – 55 goes from f to b passing through g.
   iv) NH – 44 goes from d through e to a.
   v) NH – 66 goes from g to d.

   I. Using vertices for towns and directed edges for highways between towns, draw a directed graph that models this situation.
   II. List the paths from g to a.
   III. Is it possible to leave town c and return these visiting each of the other towns only once?
   IV. What is the smallest number of highway segments that would have to be closed down in order for travel from b to d to be disrupted?
   V. What is the answer to part (3) if we are not required to return to c? **(12 Marks)**

b. Find the maximum length of i) A trial and ii) A circuit for the complete graphs K8 and K10. **(08 Marks)**

2 a. Let $X = \{1, 2, 3, 4, 5\}$ construct the loop free undirected graph $G = (V, E)$ such that,
   (V) : Let each two element subset of X represents a vertex in G.
   (E) : If $v_1, v_2 \in V$ correspond to subsets $\{a, b\}$ and $\{c, d\}$ respectively, of X, then draw the edge $\{v_1, v_2\}$ in G if $\{a, b\} \cap \{c, d\} = \phi$. **(06 Marks)**

b. Let $G = (V, E)$ be a loop free graph with $|V| = n \geq 3$. If $\deg(x) + \deg(y) \geq n$ for all non adjacent $x, y \in V$, then G contains a Hamilton cycle. **(06 Marks)**

c. For $n \geq 3$, let c_n denote the cycle of length n,
   i) What is the chromatic polynomial $P(c_3, \lambda)$?
   ii) Establish the relationships $P(c_n, \lambda) - (\lambda - 1)^n = (\lambda - 1)^{n-1} - P(c_{n-1}, \lambda)$, $P(c_n, \lambda) - (\lambda - 1)^n = P(c_{n-2}, \lambda) - (\lambda - 1)^{n-2}$ for $n \geq 5$ for $n \geq 4$.
   iii) Prove that for all $n \geq 3$, $P(c_n, \lambda) = (\lambda - 1)^n + (-1)^n (\lambda - 1)$. **(08 Marks)**

3 a. Find the depth first spanning tree for the graph shown in Fig.Q.3(a) if the order of the vertices is given as
   i) a, b, c, d, e, f, g, h
   ii) a, b, c, d, h, g, f, e **(06 Marks)**

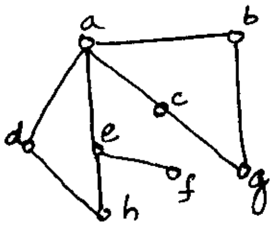

Fig.Q.3(a)

b. For every tree T = (V, E), if $|V| \geq 2$ show that T has at least two pendent vertices. **(06 Marks)**

c. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendent vertices in T. **(08 Marks)**

4 a. Apply Dijkstra's algorithm to the weighted directed multigraph shown in Fig.Q.4(a). Find the shortest distance from the vertex a to the other seven vertices in the graph. **(08 Marks)**

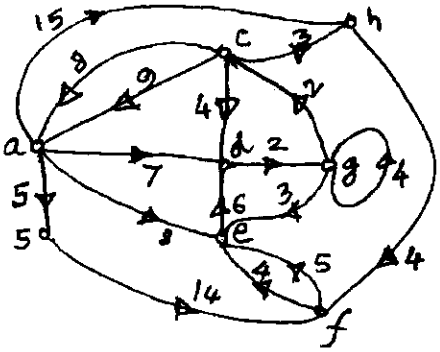

Fig.Q.4(a)

b. For the graph shown in Fig.Q.4(b), if four edges are selected at random, what is the probability that they provide a complete matching of X into Y? Where X = $\{c_1, c_2, c_3, c_4\}$ and Y = $\{s_1, s_2, s_3, s_4, s_5\}$. **(04 Marks)**

c. Define the following terms with respect to a bipartite graph with V partitioned as XUY bipartite graph.
   i) A maximal matching in G.
   ii) The deficiency of graph G.
   iii) A complete matching in G. **(04 Marks)**

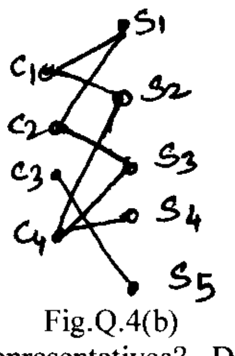

Fig.Q.4(b)

d. What is a system of distinct representatives? Determine all systems of distinct representatives for collection of sets. $A_1 = \{1, 2\}$, $A_2 = \{2, 3\}$, $A_3 = \{3, 4\}$, $A_4 = \{4, 1\}$. **(04 Marks)**

PART – B

5 a. Maruti cars in 4 models, 12 colors, 3 engine types and 2 transmission types. How many distinct Maruti cars can be manufactured? Of these how many have the same color? **(06 Marks)**

b. In how many possible ways could a student answer a 10 question TRUE/FALSE test? **(04 Marks)**

c. In how many ways can we distribute 8 identical balls into 4 distinct containers so that,
   i) No container is left empty.
   ii) The fourth container has an odd number of balls in it. **(10 Marks)**

6 a. In how many ways can 3 x's, 3 y's and 3 z's be arranged so that no consecutive triple of the same letter appears? **(06 Marks)**

b. Sheela has 7 books to review for the ABC company, so she hires 7 people to review them. She wants two reviews per book, so the first week she gives each person 1 book to read and then redistributes the books at the start of the second week. In how many ways can she make these two distributions so that she gets 2 reviews of each book? **(06 Marks)**

c. In how many ways can one distribute 10 distinct prizes among 4 students with exactly 2 students getting nothing? How many ways have at least two students getting nothing? **(08 Marks)**

7 a. Determine the coefficient of x^8 in $\dfrac{1}{(x-3)(x-2)^2}$. **(10 Marks)**

b. Find a formula to express $0^2 + 1^2 + 2^2 + \ldots\ldots + n^2$ as a function of n. **(10 Marks)**

8 a. Find a recurrence relation for the number of binary sequences of length n that have no consecutive 0s. **(06 Marks)**

b. Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$ where $n \geq 2$ and $a_0 = 1$, $a_1 = 3$. **(08 Marks)**

c. Solve the following recurrence relations by the method of generating functions $a_{n+1} - a_n = n^2$, $n \geq 0$, $a_0 = 1$. **(06 Marks)**

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